

nificant law of nature and its implications in turbulence modeling need to be further explored.

References

- ¹Busnaina, A. A., Ahmadi, G., and Chowdhury, S. J., "Two-Equation Model Consistent with Second Law," *AIAA Journal*, Vol. 25, Dec. 1987, pp. 1543-1544.
- ²Busnaina, A. A., Ahmadi, G., and Chowdhury, S. J., "Two-Equation Turbulence Model Consistent with Second Law," AIAA Paper 86-0027, Jan. 1986.
- ³Patel, V. C., Rodi, W., and Scheuerer, G., "Turbulence Models for Near-Wall and Low Reynolds Number Flows: A Review," *AIAA Journal*, Vol. 23, Sept. 1985, pp. 1308-1319.

Comment on "Stresses and Rate of Twist in Single-Cell Thin-Walled Beams with Anisotropic Walls"

Wan T. Tsai*

Rockwell International, Downey, California

BASED upon thin-walled beam assumptions, the author successfully obtained the interactive correlations between deformations of stretching, twisting, and bending, and applied loads of axial tension, torque, and bending moments for beams of anisotropic materials. Although, the conclusions were correct, the author appeared to be unaware of the earlier publications of Reissner and Tsai² and Tsai³. In these publications, a general class of anisotropic material was considered by using thin cylindrical shell theory. Closed-form solutions were obtained and effects of cross-sectional contraction were discussed. Limitation of material properties in a particular form and the assumption of neglecting the normal stress resultant N_s along the circumferential coordinate, as what were considered in the paper, were not required.

In discussing anisotropic thin-walled beams, the writer would like to share with the readers an important property that does not happen in isotropic beams nor can be determined by using the classic beam assumptions. Specifically, it is known that the stretching and bending rigidities of a beam remain unchanged between a closed cross section and the same cross section with a longitudinal slit if the material is isotropic. However, the rigidities may be significantly altered between the above-mentioned cross sections if the material is anisotropic. For orthotropically laminated thin-walled beams, the

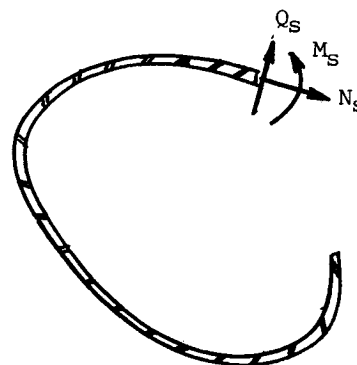


Fig. 1 Stress and moment resultants along circumferential coordinate.

difference in rigidities between a closed cross section and the same cross section with a longitudinal slit was illustrated by Tsai⁴ and Reissner and Tsai.⁵ The contributing factor to the rigidity change comes from contraction of the cross section. Specifically, due to anisotropy (such as differential lamina angles between layers) across the wall thickness, the shape of the cross section tends to deform owing to contraction between layers. Such a potential shape change will be counterbalanced by the presence of normal stress resultant N_s , shearing stress resultant Q_s , and moment resultant M_s , as shown in Fig. 1. When these resultants are neglected, as commonly assumed in classic beam theory, the cross section becomes free to deform. Accordingly, the rigidities between the cases with and without neglected N_s , Q_s , and M_s are significantly different. For the particular class of laminated thin-walled beam illustrated in Refs. 4 and 5, the rigidity of a closed cross section is about twice that of the same cross section with a longitudinal slit for an equivalent Poisson's ratio of 0.5. This reveals that 1) the commonly applied assumption of neglecting N_s , Q_s , and M_s for beams with isotropic material may not be applicable to beams with anisotropic material, and 2) the use of classic-beam assumptions may not be able to determine the correct beam rigidity; thin cylindrical shell theory is suggested for use in the computation of thin-walled beam rigidities if the material is anisotropic.

References

- ¹Libove, C., "Stresses and Rate of Twist in Single-Cell Thin-Walled Beams with Anisotropic Walls," *AIAA Journal*, Vol. 26, Sept. 1988, pp. 1107-1118.
- ²Reissner, E. and Tsai, W. T., "Pure Bending, Stretching, and Twisting of Anisotropic Cylindrical Shells," *Journal of Applied Mechanics*, Vol. 39, March 1972, pp. 148-154.
- ³Tsai, W. T., "On the Problem of Flexure of Anisotropic Cylindrical Shells," *International Journal of Solids Structures*, Vol. 9, 1973, pp. 1155-1175.
- ⁴Tsai, W. T., "On The Problems of Pure Bending, Stretching, Twisting, and Flexure of Anisotropic Cylindrical Shells," Ph.D. Dissertation, Dept. of Applied Mechanics and Engineering Sciences, Univ. of California, San Diego, CA, 1972.
- ⁵Reissner, E. and Tsai, W. T., "On Pure Bending and Stretching of Orthotropic Laminated Cylindrical Shells," *Journal of Applied Mechanics*, Vol. 41, March 1974, pp. 168-172.